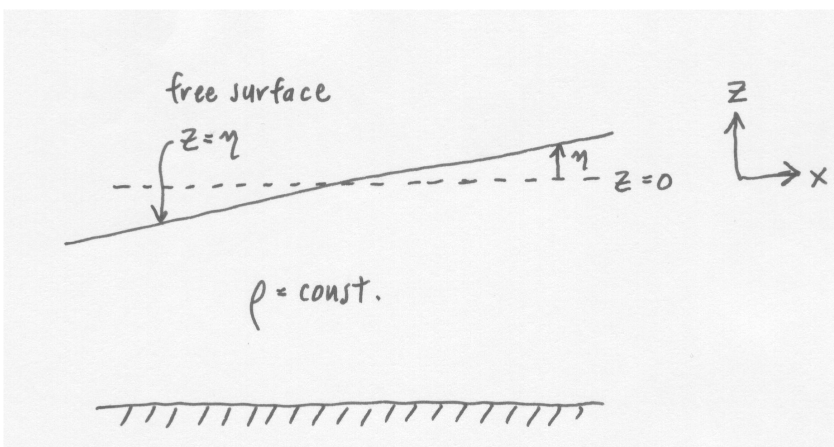
GFD I: Problem set #1, 1/13/2012

Due at the start of class on Friday 1/20/2012

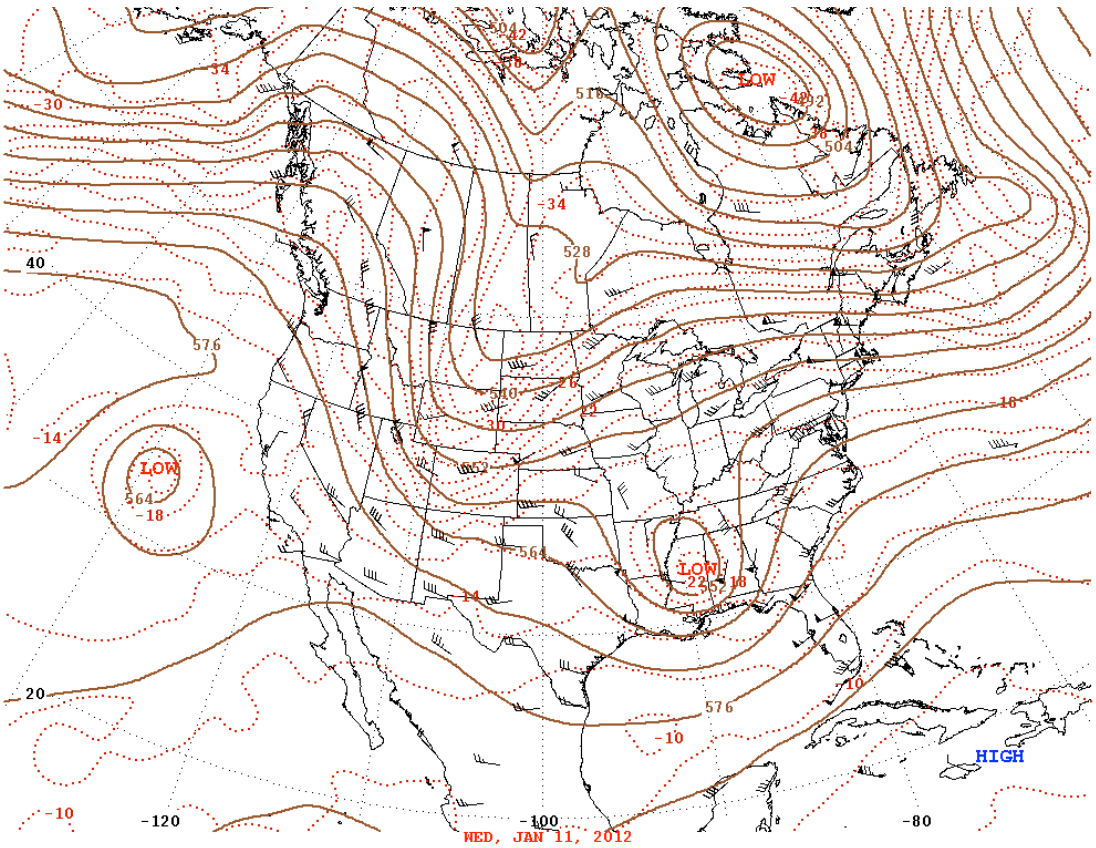
1) For homogenous water with a sloping free surface at $z = \eta$:

(a) What is the expression for $\dfrac{\partial p}{\partial x}$ in the fluid if the pressure is hydrostatic and the atmospheric pressure is constant?

(b) Assuming $\dfrac{\partial u}{\partial t} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$ [x-mom], find the x-velocity change for a fluid parcel after 1 day, if $\dfrac{\partial \eta}{\partial x} = \dfrac{1\text{ m}}{100\text{ km}}$. This is similar to the scales of the sea surface tilt across the western side of the Gulf Stream current in the Atlantic. Use $g = 9.8\text{ m s}^{-1}$ and $\rho = 1000\text{ kg m}^{-3}$.



(c) Find the same velocity change for a parcel of *air* using $\left.\frac{\partial \Phi}{\partial x}\right|_p$ or $\left.\frac{\partial \Phi}{\partial y}\right|_p$ from anywhere on the figure below. Note that the 500 mb height contours are given in 10's of meters. These are the brown lines, with numbers like 528, meaning 5280 meters above sea level. Also, there are about 111.3 km per degree of latitude.



500-Millibar Height Contours at 7:00 A.M. E.S.T.

2) In the thought problem from class we compressed an ideal gas (~dry air) in a cylinder isentropically to half its original volume. How much do ρ, p, and T increase? Express your answer as the ratio of the final to the original quantity. Explain why the fractional changes are different for all three properties. Can you think of a physical explanation for why temperature would increase? It is helpful to know that the temperature of the gas is proportional to the average kinetic energy of the molecules. You may use equations [I] and [II] from problem 3.

3) Using:

[I] $p = \rho RT$ (ideal gas law)

[II] $\dfrac{p}{\rho^{\gamma}} = \text{constant}$ (true for isentropic changes of an ideal gas)

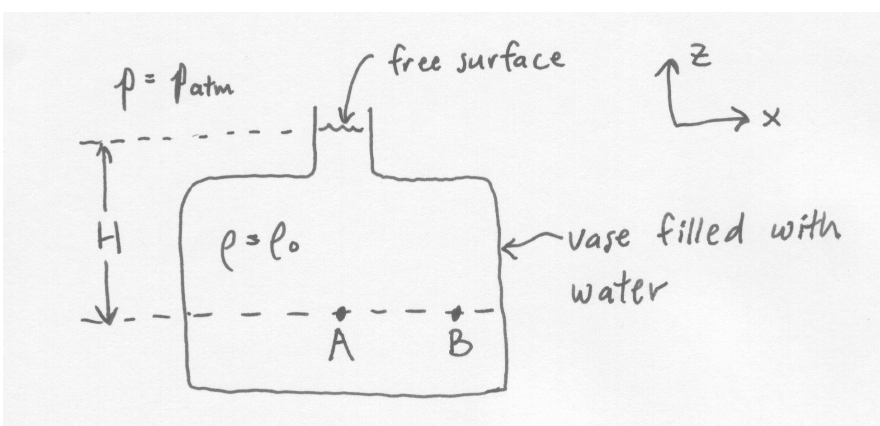
[III] $\dfrac{\partial p}{\partial z} = -\rho g$ (hydrostatic z-mom)

(a) Find $\dfrac{\partial T}{\partial z}$ for isentropic changes in height of a fluid parcel. Assume that the entire atmosphere is isentropic, with the same potential temperature. This is the "dry adiabatic lapse rate" [Holton section 2.7.2, Vallis 2.230-1].

(b) Find $p(z)$ for $T = \text{constant} = 250\ \text{K}$, with $p(z=0) = p_0$. At what height does $p = p_0 e^{-1}$? This is the "scale height" of the atmosphere [Holton section 1.6, Vallis 1.171].

NOTE: do these first yourself and then check, and correct, your answers using the texts. It is OK to derive the results in alternate ways, but the answers should be the same.

4) For a vase filled with homogenous water under the influence of gravity, what is the pressure near the bottom <u>directly under the hole at the top</u> at depth H (point A)? Now this vase, like most bottles, is wider near the bottom than at the top. What is the pressure near the side of the vase, still at depth H, but in a location that is <u>not</u> under the hole at the top (location B)? Explain your answer.



5) Sketch the hydrostatic pressure distribution and resulting acceleration field for a body of water that has constant depth, a flat free surface, which is "vertically well-mixed" (meaning that the density is vertically-constant, but may vary horizontally), and in which there is a constant increase of density in the x-direction, given by $\frac{\partial \rho}{\partial x}$. You may again assume $\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$.